Numeric Response Questions

Differential Equations

Q.1 If m and n are order and degree of the equation $\left(\frac{d^2y}{dx^2}\right)^3 + 4 \cdot \frac{\left(\frac{d^2y}{dx^2}\right)^3}{\frac{d^3y}{dx^3}} + \frac{d^3y}{dx^3} = x^2 - 1$, then find value of $m + n$.

Q.2 If the slope of the curve $y = \frac{ax}{b-x}$ at the point (1,1) is 2 , then find the value of $a + b$.

Q.3 Find the degree of differential equation $[1 + 2(y')^2]^{32} = 5y$.

Q.4 If $xdy = ydx + y^2dy$ and $y(1) = 1$ then find the value of $y(-3)$.

Q.5 If $\phi(x) = \phi'(x), \phi(1) = 2$ and $\phi(3) = ke^2$ then find k.

Q.6 Equation of the curve passing through the point (1,2) such that the intercept on the x-axis cut off between the tangent and origin is twice the abacissa is given by $xy = k$ then find k.

Q.7 Find the order of the differential equation, whose general solution is $y = c_1e^2 + c_2e^{2\,s} + c_2e^{3x} + c_4e^{x+\varepsilon_3}$, (where c_1, c_2, c_3, c_4, c_5 are arbitrary constants)

Q.8 Find the sum of order and degree of differential equation of all tangent lines to the parabola $x^2 = 4y$.

Q.9 Equation of curve through (2,2) satisfying $(1 - x^2)\frac{dy}{dx} + xy = 5x$ is $(y - 5)^2 + k(1 - x^2) = 0$ then find k.

Q.10 The slope at any point of a curve $y = f(x)$ is given by $\frac{dy}{dx} = 3x^2$ and it passes through $(-1,1)$. The equation of the curve is $y = x^n + 2$ then find n.

Q.11 The solution of $\frac{dy}{dx} + \frac{y}{(1+x^2)} = \frac{e^{\tan^{-1} x}}{1+x^2}$ is $e^{k\tan^{-1} x} + 2c$ then find k.

Q.12 The solution of the differential equation $(2x - 10y^3)\frac{dy}{dx} + y = 0$ is $xy'' = 2y^m + C$ then find $n + m$.

Q.13 The solution of $\frac{dy}{dx} = \frac{6x^2}{(2y+\cos y)}, y(1) = 0$, is $y^2 + \sin\, y = kx^3 - 2$ then find k.

Q.14 If the integral factor of equation $(x^2 + 1)\frac{dy}{dx} + 2xy = x^2 - 1$, is $x^2 + k$ then find k.

Q.15 The equation of the curve through the point (3,2) and whose slope is $\frac{x^2}{y+1}$, is $\frac{y^2}{2} + y = \frac{x^3}{3} - k$ then find k.

ANSWER KEY

1. 5.00	**2.** 3.00	**3.** 2.00	**4.** 3.00	**5.** 2.00	**6.** 2.00	**7.** 3.00
8. 3.00	**9.** 3.00	**10.** 3.00	**11.** 2.00	**12.** 7.00	**13.** 2.00	**14.** 1.00
15. 5.00						

Hints & Solutions

1. The given differential equation can be written as

$$\left(\frac{d^2y}{dx^2}\right)^5 \frac{d^3y}{dx^3} + 4\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{d^3y}{dx^3}\right)^2$$

$$= (x^2 - 1)\frac{d^3y}{dx^3}$$

$\Rightarrow m = 3, n = 2$

2. We have, $y = \frac{ax}{b-x}$

$$\Rightarrow \frac{dy}{dx} = \frac{(b-x)a - ax.(-1)}{(b-x)^2} = \frac{ab}{(b-x)^2}$$

$$\therefore \left[\frac{dy}{dx}\right]_{(1,1)} = \frac{ab}{(b-1)^2} = 2 \text{ (given)} \qquad ....(i)$$

Since the curve passes through the point (1, 1), therefore,

$$1 = \frac{a}{b-1} \Rightarrow a = b - 1$$

On putting $a = b - 1$ in equation (i), we get

$$\frac{(b-1)b}{(b-1)^2} = 2 \Rightarrow b = 2. \quad \therefore \quad a = 2 - 1 = 1$$

Hence, $a = 1$, $b = 2$

3. Do yourself

4. $$\int \frac{y\,dx - x\,dy}{y^2} = \int -dy$$

$$\frac{x}{y} = -y + c$$

As, $y(1) = 1$

$1 = -1 + c$

$\Rightarrow c = 2$

$\Rightarrow \frac{x}{y} = -y + 2$

If $x = -3$

$-3 = -y^2 + 2y$

$\Rightarrow y^2 - 2y - 3 = 0$

$\Rightarrow y = 3$

5. $\phi(x) = \phi'(x) \Rightarrow \frac{\phi'(x)}{\phi(x)} = 1$

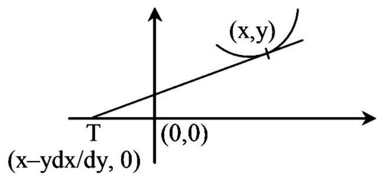
$\Rightarrow \log \phi(x) = x + \log C \Rightarrow \phi(x) = Ce^x$

Putting $x = 1$, $\phi(1) = 2$, we get $C = \frac{2}{e}$

$\therefore \phi(x) = 2e^{x-1} \Rightarrow \phi(3) = 2e^2$

6. equation of tangent

$$Y - y = \frac{dy}{dx}(X - x)$$



$$\because \; x - y\frac{dx}{dy} = 2x$$

$$\int \frac{dy}{y} = -\int \frac{dx}{x}$$

7. Rewriting the given general solution, we have

$$y = c_1e^x + c_2e^{2x} + c_3e^{3x} + c_4e^x.e^{c_5}$$

$$= (c_1 + c_4.e^{c_5})e^x + c_2e^{2x} + c_3e^{3x}$$

$$= c_1'e^x + c_2e^{2x} + c_3e^{3x}$$

Where $c_1' = c_1 + c_4.e^{c_5}$.

So there are 3 arbitrary constant associated with different terms. Hence the order of the differential equation formed, will be 3.

8. $y = mx - am^2$

$\Rightarrow y = mx - m^2$

$\Rightarrow y' = m$

$\Rightarrow y = y'x - (y')^2$

9. $\frac{dy}{dx} + \frac{x}{1-x^2} y = \frac{5x}{1-x^2}$ whose solution is

$y.e^{\int \frac{x^2}{1-x^2} dx} = \int \left(\frac{5x}{1-x^2}\right) \text{I.F. } dx + c$

10. $\frac{dy}{dx} = 3x^2$

$y = 3\frac{x^3}{3} + c \quad \Rightarrow y = x^3 + c$

It passes through (–1, 1)

$1 = -1 + c \quad \Rightarrow c = 2$

Hence equation of curve is $y = x^3 + 2$

11. $\frac{dy}{dx} + \frac{y}{(1+x^2)} = \frac{e^{\tan^{-1}x}}{1+x^2}$

Multiplying both sides I.F. and integrating

$ye^{\tan^{-1}x} = \int \frac{e^{\tan^{-1}x}}{(1+x^2)} e^{\tan^{-1}x} dx + c$

Put $e^{\tan^{-1}x} = t, \quad \frac{e^{\tan^{-1}x}}{1+x^2} dx = dt$

$\Rightarrow ye^{\tan^{-1}x} = \int t\, dt + c = \frac{t^2}{2} + c$

$\Rightarrow 2\, ye^{\tan^{-1}x} = e^{2\tan^{-1}x} + 2c$

12. $y\frac{dx}{dy} + 2x = 10y^3$

$\Rightarrow \frac{dx}{dy} + \frac{2x}{y} = 10y^2$

$\text{I.F.} = e^{\int \frac{2}{y} dy} = y^2$

$x.y^2 = \int 10y^4 dy$

$\Rightarrow xy^2 = 2y^5 + c$

13. $\int (2y + \cos y)\, dy = \int 6x^2\, dx$

14. Given equation may be written as

$\frac{dy}{dx} + \frac{2x}{x^2+1} y = \frac{x^2-1}{x^2+1}$

Comparing with $\frac{dy}{dx} + Py = Q$, $P = \frac{2x}{x^2+1}$

$\text{I.F.} = e^{\int P dx} = e^{\int \frac{2x dx}{1+x^2}} = e^{\ell n(1+x^2)} = 1 + x^2$

15. $\frac{dy}{dx} = \frac{x^2}{y+1}$

$\Rightarrow \int (y+1) dy = \int x^2 dx$

$\Rightarrow \frac{y^2}{2} + y = \frac{x^3}{3} + C$

It passes through (3, 2) so c = – 5

$\therefore$ Required curve is $\frac{y^2}{2} + y = \frac{x^3}{3} - 5$